4. Exercise on Network Coding Theory

Problem 6:

We consider the well-known butterfly network and assume that each transmission over an edge is delayed by a normalized delay of $D$. Further, we denote by $\sigma_1(t)$ and $\sigma_2(t)$ the symbols that are transmitted by the two source nodes at time $t, \geq 1$. At the network coding point (node 3), the incoming symbols are combined by an XOR operation.

![Butterfly network with delay](image)

**Figure 3:** Butterfly network with delay

a) Describe the network with the global coding vectors at the two receivers, i.e., the matrices $A_1$ and $A_2$ as a function of $D$,

\[
y_1(D) = A_1(D)\sigma(D)
y_2(D) = A_2(D)\sigma(D)
\]

with $\sigma_1(D) = \sum_t \sigma_1(t)D^t$ and $\sigma_2(D)$ analogue.

b) Draw the block diagram of the butterfly network with delay.

c) After how many time slots is the receiver $R_1$ able to decode the symbols $\sigma_1(t)$ and $\sigma_2(t)$? How does the data processing for the decoding look like?
Problem 7:
Consider the following network with 2 sources and 3 receivers.

![Network Diagram]

**Figure 4:** Network with $h = 2$ sources and $N = 3$ receivers

**Hint:** This network is no longer acyclic. With the following assumptions we can treat the network as usual: Node F forwards the packets from node A and keeps the packets from node H.

a) Determine the min-cut max-flow value from the sources to all receivers.
b) Draw the line graph of the network.
c) Do a minimal sub-tree decomposition of the network.
d) Determine the matrices $A, B, C_1, C_2, C_3$ as well as $D_1, D_2, D_3$ of the state space equations. Further, calculate the global encoding matrices $A_1, A_2,$ and $A_3$ at nodes E, F and K depending on the variables $\{\alpha_k\}$.
e) Solve the network coding problem and provide the receiver matrices $A_1, A_2,$ and $A_3$ over the smallest possible field $\mathbb{F}_q$. 